

"O day and night, but this is wondrous strange"



No Dimensions
POINTLAND

Two Dimensions
FLATLAND

One Dimension
LINELAND

Three Dimensions
SPACELAND

"Fie, fie, how frantically I square my talk!"

FLATLAND

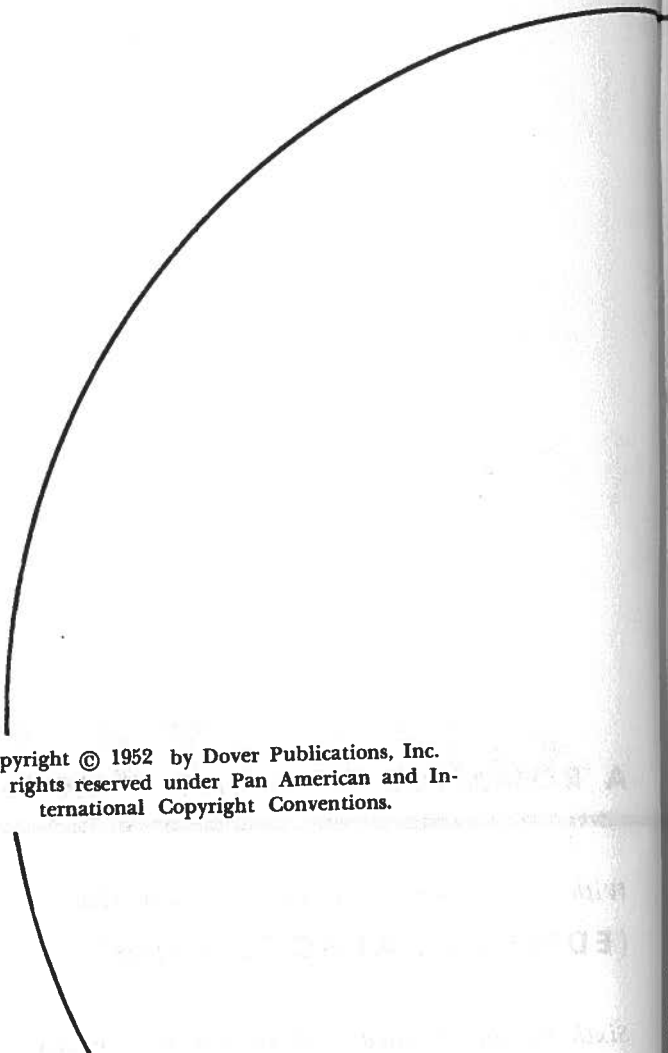
A ROMANCE OF MANY DIMENSIONS

With Illustrations by the Author, A SQUARE

(EDWIN A. ABBOTT 1838-1926)

Sixth Edition, Revised With Introduction by BANESH HOFFMANN

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To

The Inhabitants of SPACE IN GENERAL
And H. C. IN PARTICULAR
This Work is Dedicated
By a Humble Native of Flatland
In the Hope that
Even as he was Initiated into the Mysteries
Of THREE Dimensions
Having been previously conversant
With ONLY Two
So the Citizens of that Celestial Region
May aspire yet higher and higher
To the Secrets of FOUR FIVE OR EVEN SIX Dimensions
Thereby contributing
To the Enlargement of THE IMAGINATION
And the possible Development
Of that most rare and excellent Gift of MODESTY
Among the Superior Races
Of SOLID HUMANITY

INTRODUCTION

Here is a stirring adventure in pure mathematics, a fantasy of strange spaces peopled by geometrical figures; geometrical figures that think and speak and have all too human emotions. This is no trifling tale of science fiction. Its aim is to instruct, and it is written with subtle artistry. Start it and you will fall under its spell. If you are young in heart and the sense of wonder still stirs within

you, you will read without pause till the end is regretfully reached. Yet you will not guess when the tale was written nor by what manner of man.

In these days space-time and the fourth dimension are household words. But Flatland, with its vivid picture of one and two and three and more dimensions, was not conceived in the era of relativity. It was written some seventy years ago, when Einstein was a mere child and the idea of space-time lay almost a quarter of a century in the future.

In those far off days, to be sure, the professional mathematicians were imagining spaces of any number of dimensions. The physicists too, in their theorizing, were working with hypothetical graph-spaces of arbitrary dimensionality. But these were matters of abstract theory. There was no public clamor for their elucidation; the public hardly knew that they existed.

One would think, therefore, that, in order to write Flatland, Edwin A. Abbott must have been a mathematician or physicist. But he was neither of these. True, he was a schoolmaster—a headmaster, no less, and a most distinguished one. But his field was classics, and his primary interests literature and theology, on which he wrote several books. Does this sound like the sort of man who would write an absorbing mathematical adventure? Perhaps Abbott himself thought it did not, for he published Flatland pseudonymously, as if afraid that it might besmirch the dignity of his more formal writings, of which he betrayed no reluctance to acknowledge his authorship.

Much has happened to our ideas of space and time since Flatland came into being. But despite all the talk of a fourth dimension, the fundamentals of dimensionality have not changed. Long before the advent of the theory of relativity, scientists thought of time as an extra dimension. In those days they regarded it as a solitary, isolated dimension that kept aloof from the three dimensions of space. In relativity time became inextricably intermingled

with space to form a truly four-dimensional world; and this four-dimensional world turned out to be a curved one.

These modern developments have less significance than one might imagine for the story of Flatland. We do indeed have four dimensions. But even in relativity, they are not all of the same sort. Only three are spatial. The fourth is temporal; and we are unable to move freely in time. We can not return to days gone by, nor avoid the coming of tomorrow. We can neither hasten nor retard our journey into the future. We are like hapless passengers on a crowded escalator, carried relentlessly forward till our particular floor arrives and we step off into a place where there is no time, while the material composing our bodies continues its journey on the inexorable escalator—perhaps forever.

Time, the tyrant, holds sway in Flatland as in our own world. Relativity or no relativity, we still have only one dimension more than the creatures of Abbott's imagination; we still have only three spatial dimensions to their two. The inhabitants of Flatland are sentient beings, troubled by our troubles and moved by our emotions. Flat they may be physically, but their characters are well rounded. They are our kin, our own flesh and blood. We romp with them in Flatland. And romping, we suddenly find ourselves looking anew at our own humdrum world with the wide-eyed wonder of youth.

In Flatland we could escape from a two-dimensional prison by stepping momentarily into the third dimension and coming back on the other side of the prison wall. But that is because this third dimension is spatial. Our fourth dimension, time, true dimension though it be, does not permit us to escape from a three-dimensional prison. It does enable us to get out, for if we wait patiently for time to pass, our sentence will be served and we shall be set free. That is hardly an escape, however. To escape we must travel through time to some moment when the prison is wide open, or in ruins, or not yet built; and then, having stepped outside, we

must reverse the direction of our time travel to return to the present. Neither we nor the inhabitants of Flatland can travel thus through time.

Though the crowded years go by, this nigh on seventy year old tale shows no sign of age. It remains as spry as ever, a timeless classic of perennial fascination that seems to have been written for today. Like all great art, it defies the tyrant Time.

Banesh Hoffmann

**PREFACE TO THE SECOND
AND REVISED EDITION, 1884.
BY THE EDITOR**

If my poor Flatland friend retained the vigour of mind which he enjoyed when he began to compose these Memoirs, I should not now need to represent him in this preface, in which he desires, firstly, to return his thanks to his readers and critics in Spaceland, whose appreciation has, with unexpected celerity, required a

second edition of his work; secondly, to apologize for certain errors and misprints (for which, however, he is not entirely responsible); and, thirdly, to explain one or two misconceptions. But he is not the Square he once was. Years of imprisonment, and the still heavier burden of general incredulity and mockery, have combined with the natural decay of old age to erase from his mind many of the thoughts and notions, and much also of the terminology, which he acquired during his short stay in Spaceland. He has, therefore, requested me to reply in his behalf to two special objections, one of an intellectual, the other of a moral nature.

The first objection is, that a Flatlander, seeing a Line, sees something that must be thick to the eye as well as long to the eye (otherwise it would not be visible, if it had not some thickness); and consequently he ought (it is argued) to acknowledge that his countrymen are not only long and broad, but also (though doubtless in a very slight degree) thick or high. This objection is plausible, and, to Spacelanders, almost irresistible, so that, I confess, when I first heard it, I knew not what to reply. But my poor old friend's answer appears to me completely to meet it.

"I admit," said he—when I mentioned to him this objection—"I admit the truth of your critic's facts, but I deny his conclusions. It is true that we have really in Flatland a Third unrecognized Dimension called 'height,' just as it is also true that you have really in Spaceland a Fourth unrecognized Dimension, called by no name at present, but which I will call 'extra-height'. But we can no more take cognizance of our 'height' than you can of your 'extra-height'. Even I—who have been in Spaceland, and have had the privilege of understanding for twenty-four hours the meaning of 'height'—even I cannot now comprehend it, nor realize it by the sense of sight or by any process of reason; I can but apprehend it by faith.

"The reason is obvious. Dimension implies direction, implies measurement, implies the more and the less. Now, all our lines are equally and infinitesimally thick (or high, whichever you like);

consequently, there is nothing in them to lead our minds to the conception of that Dimension. No 'delicate micrometer'—as has been suggested by one too hasty Spaceland critic—would in the least avail us; for we should not know what to measure, nor in what direction. When we see a Line, we see something that is long and bright; brightness, as well as length, is necessary to the existence of a Line; if the brightness vanishes, the Line is extinguished. Hence, all my Flatland friends—when I talk to them about the unrecognized Dimension which is somehow visible in a Line—say, 'Ah, you mean brightness': and when I reply, 'No, I mean a real Dimension,' they at once retort 'Then measure it, or tell us in what direction it extends'; and this silences me, for I can do neither. Only yesterday, when the Chief Circle (in other words our High Priest) came to inspect the State Prison and paid me his seventh annual visit, and when for the seventh time he put me the question, 'Was I any better?' I tried to prove to him that he was 'high,' as well as long and broad, although he did not know it. But what was his reply? 'You say I am "high"; measure my "high-ness" and I will believe you.' What could I do? How could I meet his challenge? I was crushed; and he left the room triumphant.

"Does this still seem strange to you? Then put yourself in a similar position. Suppose a person of the Fourth Dimension, condescending to visit you, were to say, 'Whenever you open your eyes, you see a Plane (which is of Two Dimensions) and you infer a Solid (which is of Three); but in reality you also see (though you do not recognize) a Fourth Dimension, which is not colour nor brightness nor anything of the kind, but a true Dimension, although I cannot point out to you its direction, nor can you possibly measure it.' What would you say to such a visitor? Would not you have him locked up? Well, that is my fate: and it is as natural for us Flatlanders to lock up a Square for preaching the Third Dimension, as it is for you Spacelanders to lock up a Cube for preaching the Fourth. Alas, how strong a family like-

ness runs through blind and persecuting humanity in all Dimensions! Points, Lines, Squares, Cubes, Extra-Cubes—we are all liable to the same errors, all alike the Slaves of our respective Dimensional prejudices, as one of your Spaceland poets has said—

*'One touch of Nature makes all worlds akin.'*¹

On this point the defence of the Square seems to me to be impregnable. I wish I could say that his answer to the second (or moral) objection was equally clear and cogent. It has been objected that he is a woman-hater; and as this objection has been vehemently urged by those whom Nature's decree has constituted the somewhat larger half of the Spaceland race, I should like to remove it, so far as I can honestly do so. But the Square is so unaccustomed to the use of the moral terminology of Spaceland that I should be doing him an injustice if I were literally to transcribe his defence against this charge. Acting, therefore, as his interpreter and summarizer, I gather that in the course of an imprisonment of seven years he has himself modified his own personal views, both as regards Women and as regards the Isosceles or Lower Classes. Personally, he now inclines to the opinion of the Sphere that the Straight Lines are in many important respects superior to the Circles. But, writing as a Historian, he has identified himself (perhaps too closely) with the views generally adopted by Flatland, and (as he has been informed) even by Spaceland, Historians; in whose pages (until very recent times) the destinies of Women and of the masses of mankind have seldom been deemed worthy of mention and never of careful consideration.

In a still more obscure passage he now desires to disavow the Circular or aristocratic tendencies with which some critics have

¹The Author desires me to add, that the misconception of some of his critics on this matter has induced him to insert in his dialogue with the Sphere, certain remarks which have a bearing on the point in question, and which he had previously omitted as being tedious and unnecessary.

naturally credited him. While doing justice to the intellectual power with which a few Circles have for many generations maintained their supremacy over immense multitudes of their countrymen, he believes that the facts of Flatland, speaking for themselves without comment on his part, declare that Revolutions cannot always be suppressed by slaughter, and that Nature, in sentencing the Circles to infecundity, has condemned them to ultimate failure—"and herein," he says, "I see a fulfilment of the great Law of all worlds, that while the wisdom of Man thinks it is working one thing, the wisdom of Nature constrains it to work another, and quite a different and far better thing." For the rest, he begs his readers not to suppose that every minute detail in the daily life of Flatland must needs correspond to some other detail in Spaceland; and yet he hopes that, taken as a whole, his work may prove suggestive as well as amusing, to those Spacelanders of moderate and modest minds who—speaking of that which is of the highest importance, but lies beyond experience—decline to say on the one hand, "This can never be," and on the other hand, "It must needs be precisely thus, and we know all about it."

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